Autopilot Analysis and EP Scheme for the Twin Otter under Iced Conditions.

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Objectives

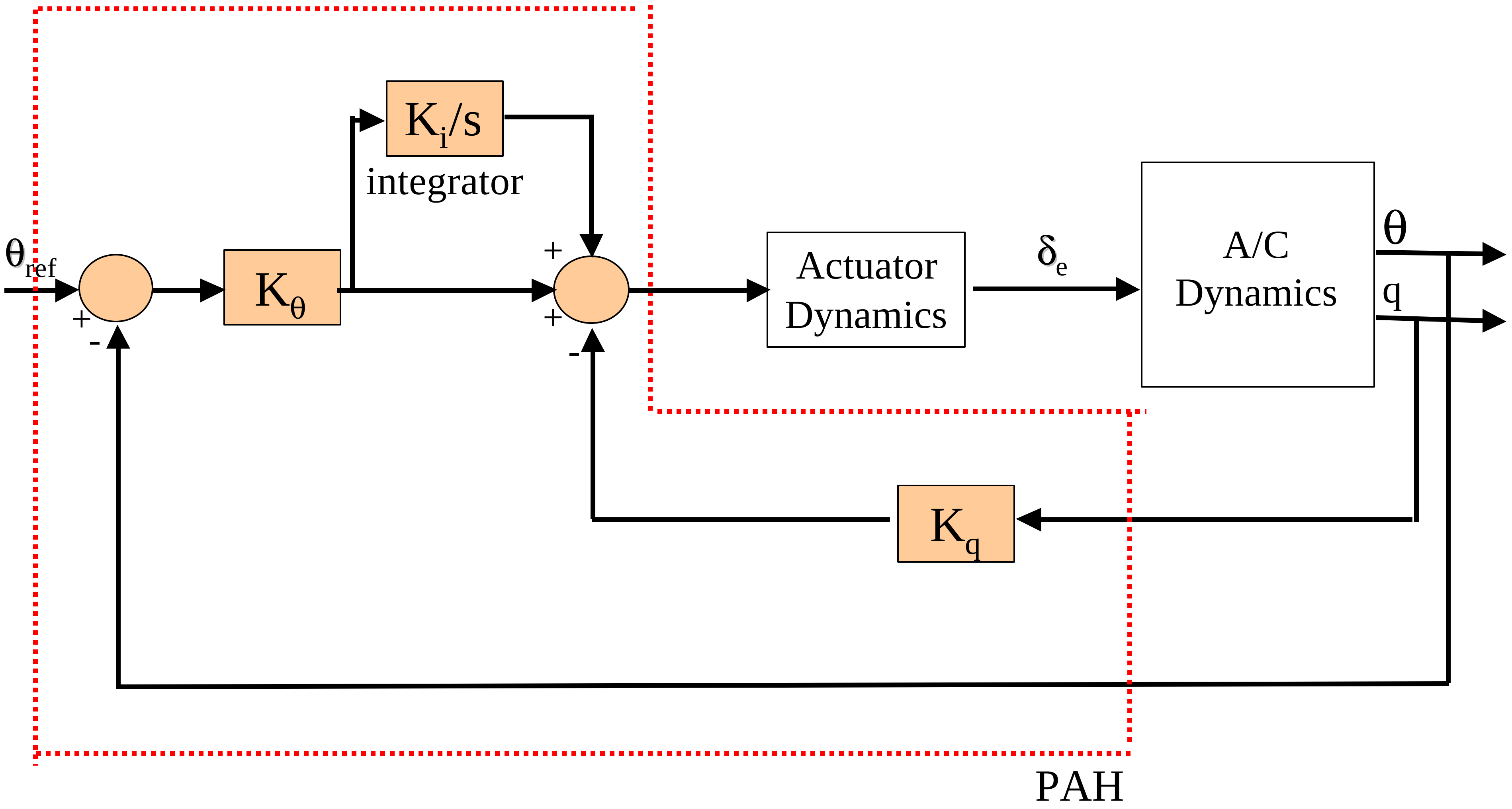
- Investigate the autopilot behavior under iced conditions.

- Develop an envelope protection scheme for the autopilot system to operate under iced conditions.

Overview of the talk

- Pitch attitude hold (PAH) autopilot (A/P) structure
- Stability analysis of the PAH A/P
- Reference value limits through Linear Matrix Inequalities analysis
- Envelope Protection scheme
- Work in progress

Block Diagram for PAH



Affine Model of Closed Loop PAH

- The closed loop model is affinely dependent on the icing parameter h, i.e.

$$\dot{x} = A(h)x$$

A is a function of *h* and can be written in the form:

$$A(h) = A_0 + h(t)A_1$$

Where $h \in \Delta$, with $\Delta = [0, h_{max}]$

Quadratic Stability

- The PAH A/P system given by

$$\dot{x} = A(\boldsymbol{h})x$$

is stable iff ∃ $K=K^T > 0$ s.t.

$$A(h)^T K + KA(h) < 0$$

For all $h \in [0, h_{max}]$

PROBLEM: This places an infinite number of constraints on the symmetric matrix *K*

Quadratic Stability

SOLUTION

- Since the system is affinely dependent on h the quadratic stability condition is the same as the following TWO LMIs

$$A(h=0)^T K + KA(h=0) < gI$$

and

$$A(h=h_{max})^T K + KA(h=h_{max}) < gI$$

where g is a negative scalar and

$$h(t) \in [0, h_{max}]$$

- The above can be solved to obtain minimum possible values of g using LMILAB in MATLAB

Linear Matrix Inequalities (LMI's)

- What are they?
- An expression of the form:

$$F(x) = F_0 + x_1 F_1 + \ldots + x_m F_m > 0$$

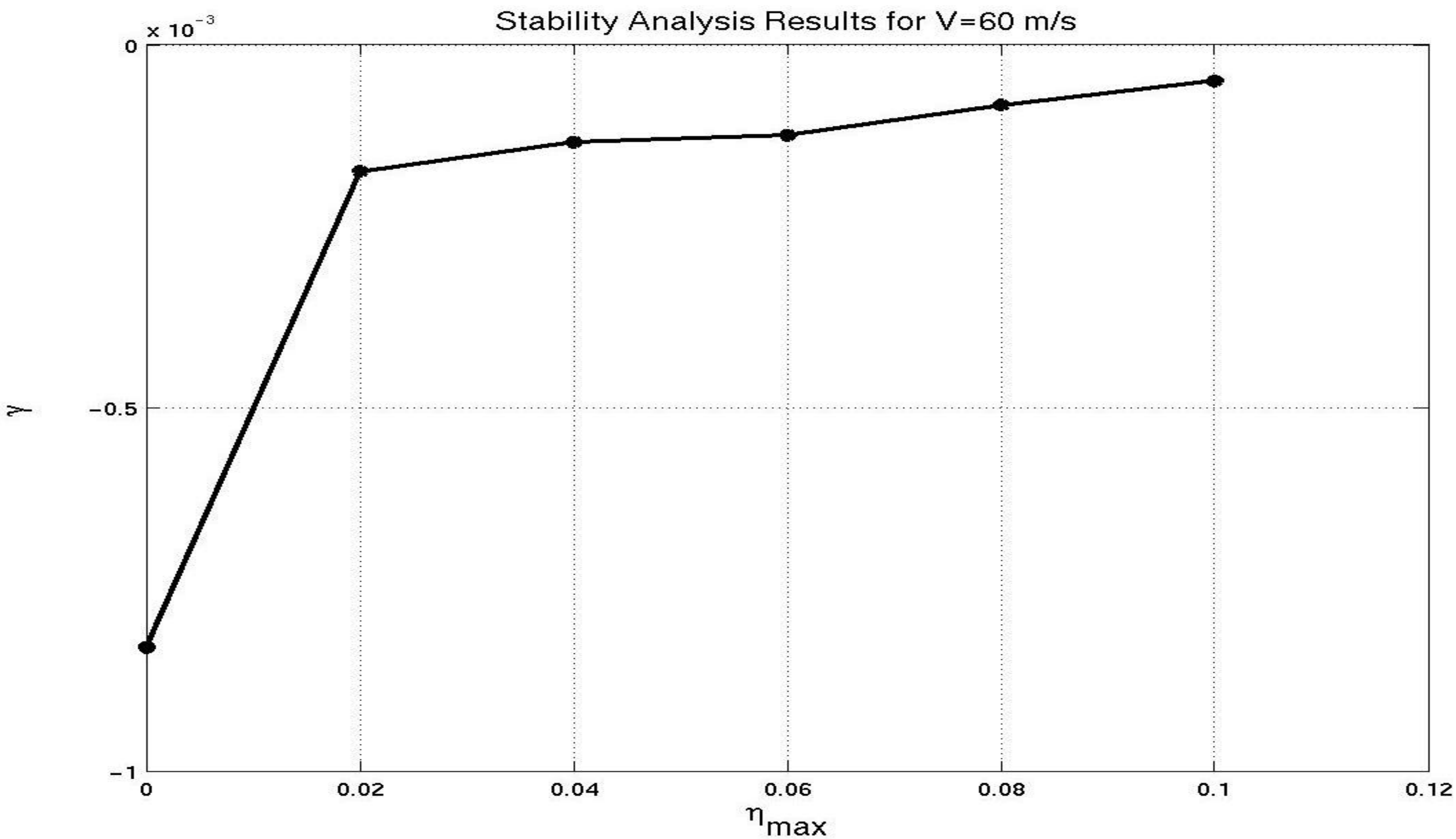
where

$X = (x_1, \ldots\ldots, x_m)$ is a vector of decision variables

$F_0, \ldots\ldots, F_m$ are real symmetric matrices,

Inequality > 0 means positive definite

Results

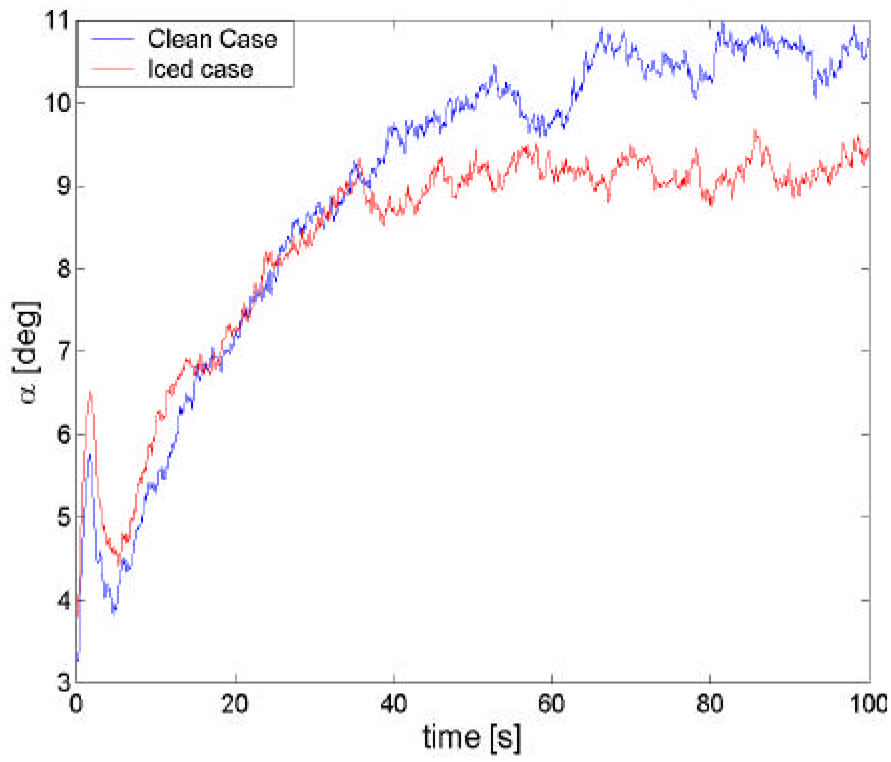

Stability Analysis Results for V=60 m/s
× 10⁻³
0
−0.5
−1
γ
0
0.02
0.04
0.06
0.08
0.1
0.12
ηmax

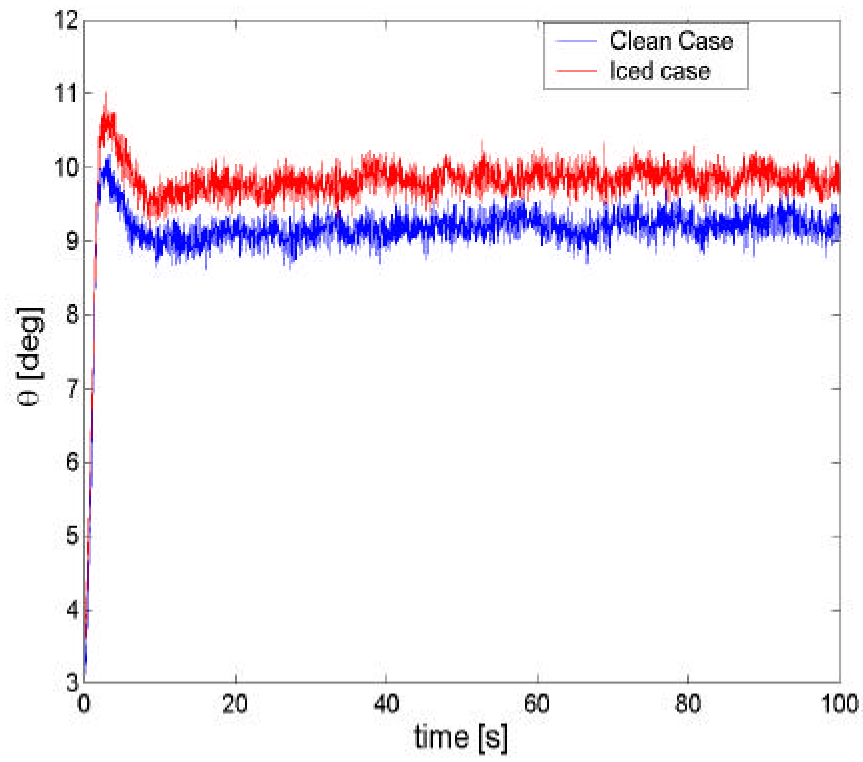
Conclusion

- The linear analysis show that the Pitch Attitude hold A/P will maintain stability under icing
- There is a small degradation in the guaranteed stability level

Simulation results

- Comparison of the Pitch Hold PID controller response at V = 60 m/s to a pitch up by 6 degrees.





Envelope Protection for PAH Autopilot

PROBLEM: Insure

$$a(t) < a_{\max}(h(t))$$

For all time

APPROACH: Keep $q_{ref}(t)$ low enough so that the above is insured

Limits on the Reference Pitch Using LMI Techniques

- Can find smallest g s.t. if $\left\| q_{ref} \right\| < 1/g$ then $|a(t)| < a_{\max}$, $|d_e(t)| < d_{e\max}$
- Values obtained using $|a_{\max}| = 18.1$ degrees for the clean and $|a_{\max}| = 13$ degrees for iced case and $|d_{e\max}| = 26$ degrees

Clean Case

Velocity (m/s)	Maximum $\lvert q_{ref}(t) \rvert$ allowed (deg)
60	6.0311
65	7.5389
70	5.8465

Iced Case (*h*=0.1)

Velocity (m/s)	Maximum $\lvert q_{ref}(t) \rvert$ allowed (deg)
60	4.5837
65	5.7296
70	5.8465

An Alternate Envelope Protection Scheme

- Earlier LMI technique might be conservative.
- Look at step pilot inputs
- Look at steady state response of the angle of attack

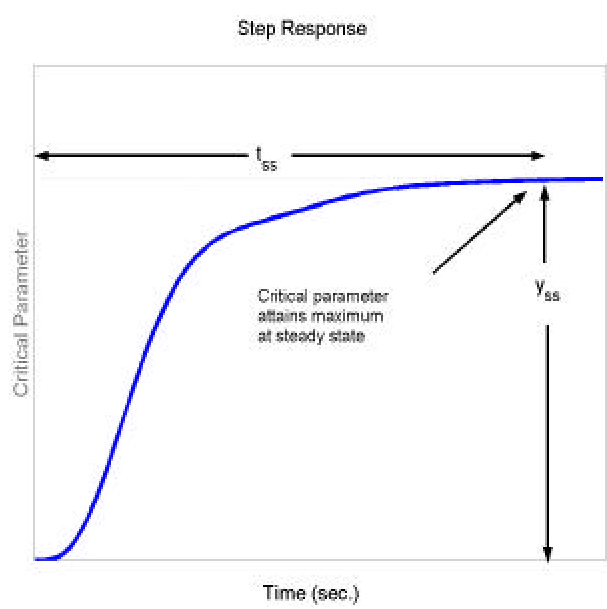
Step Response	Envelope Protection System
	***Steady State Estimation*** LIMIT DETECTION – Estimate value of limited parameter in steady state LIMIT AVOIDANCE – Find control value that causes the limited parameter to reach envelope limit in steady state

Table of limits on θ_{ref} (Clean Case) from SS and LMI approach (α_{max}=18.1 deg)

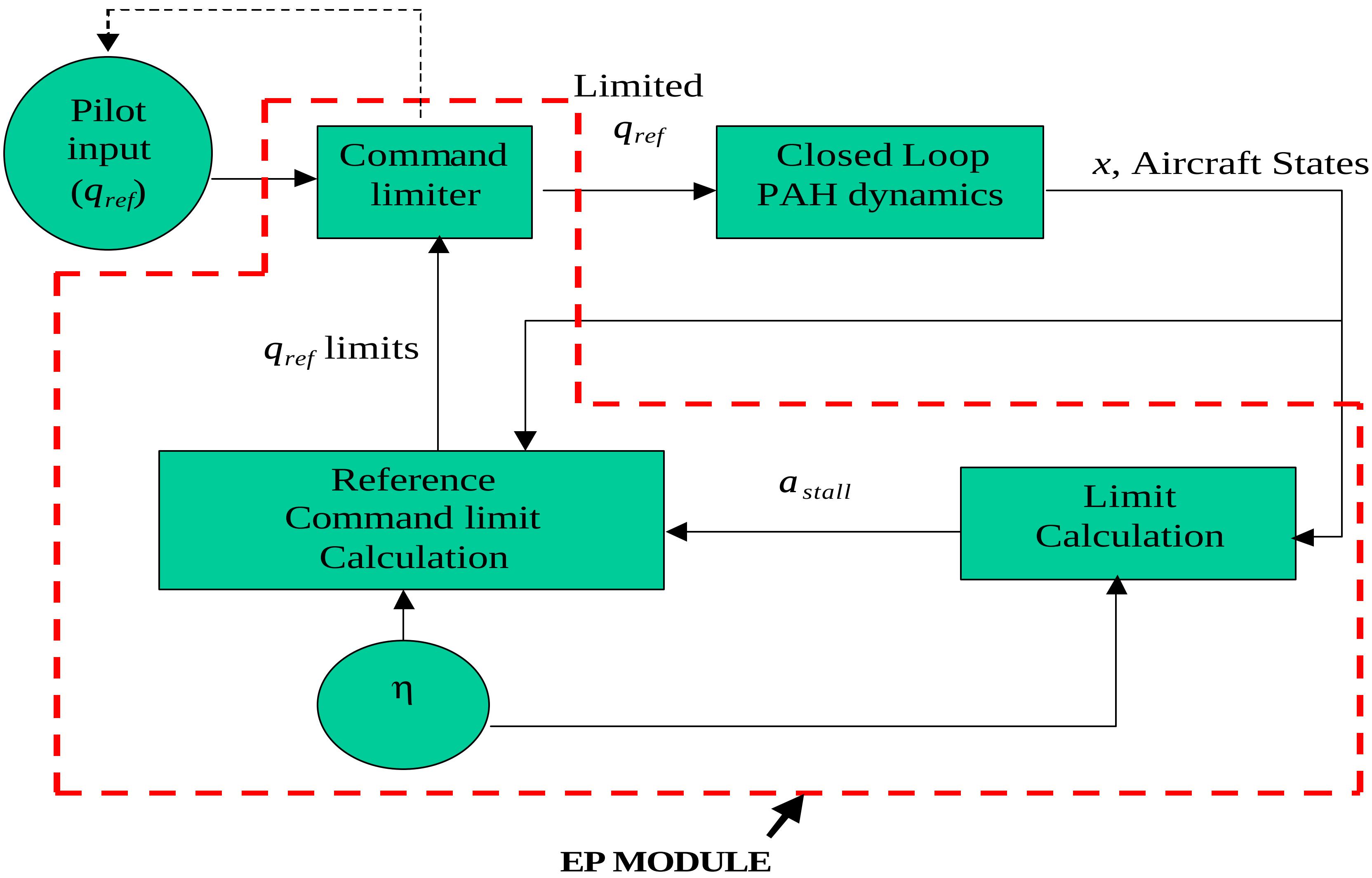
Steady State

Velocity (m/s)	Maximum step θ_{ref} allowed (deg)
60	9.4
65	10.9
70	12.6

LMI approach

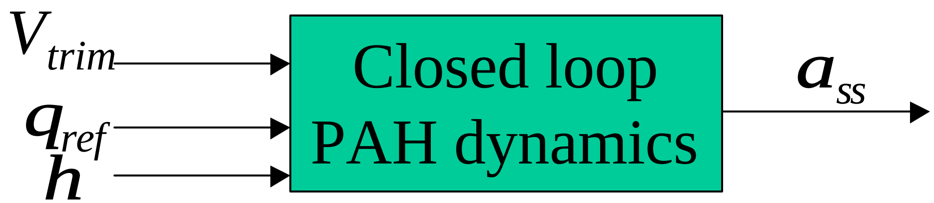
Velocity (m/s)	Maximum θ_{ref} allowed (deg)
60	6.0311
65	7.5389
70	5.8465

PAH A/P with EP Module



Data Generation

- Data is generated by issuing a range of reference pitch commands at different flying conditions



- Steady state angle of attack values corresponding to trim state values of *V*, *h* and q_{ref} are recorded

EP Module Coding Scheme

EVERY 5 SECONDS

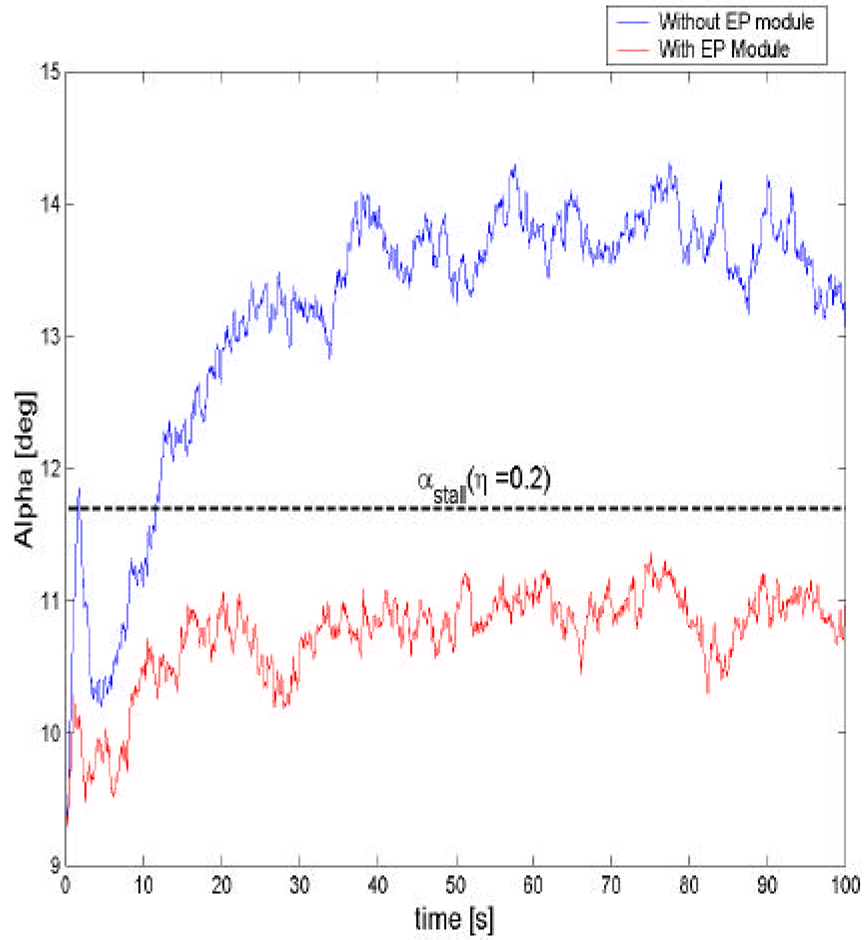
- Treat the state reached as a trim state
- Use the data generated to obtain maximum allowable q_{ref} ($q_{ref}^{\max}$) at that state

$$a_{ss} = f(V, h, q_{ref}^{\max}) \approx a_{stall}(h)$$

- Compare q_{ref} at the current point with the $q_{ref}^{\max}$ value and pitch down if necessary

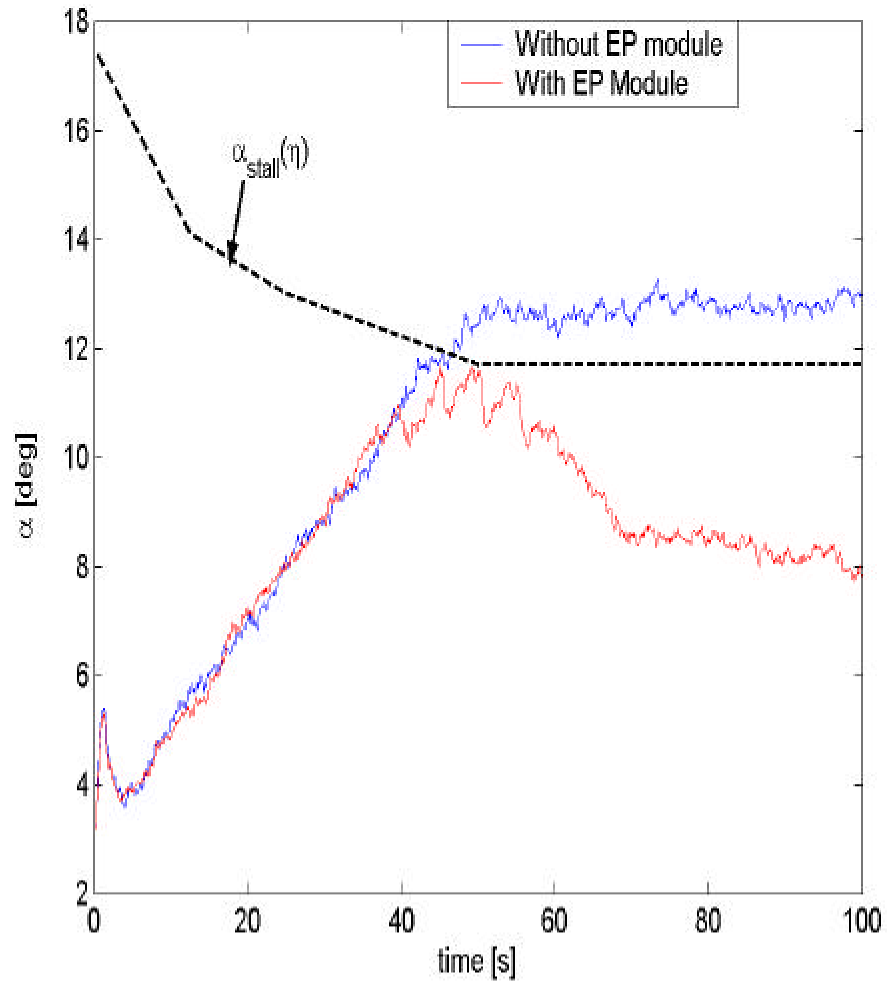
A scenario

- A/C trimmed at V = 50 m/s with η=0.2 at H=2300m.
- A pitch up command of 4 degrees issued.



Another Scenario

- A pitch up command of 4 degrees with V=60m/s is issued and ice starts to build and grows from η=0 at t=0 to η=0.2 at t=50 s.



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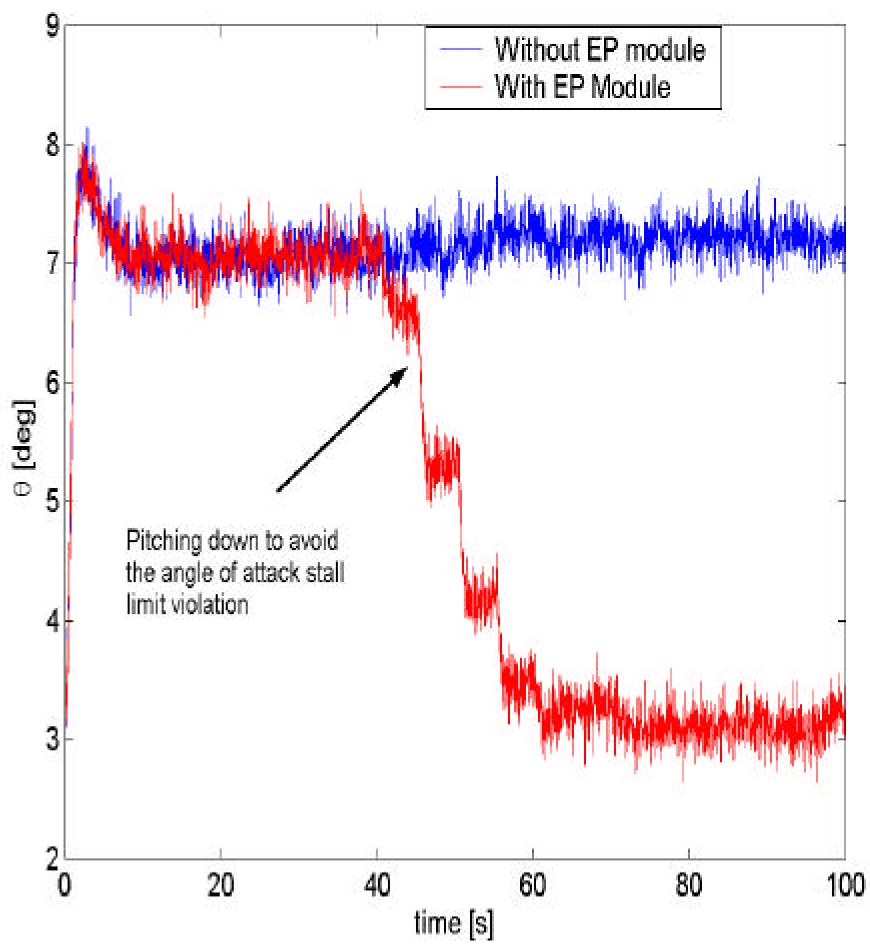
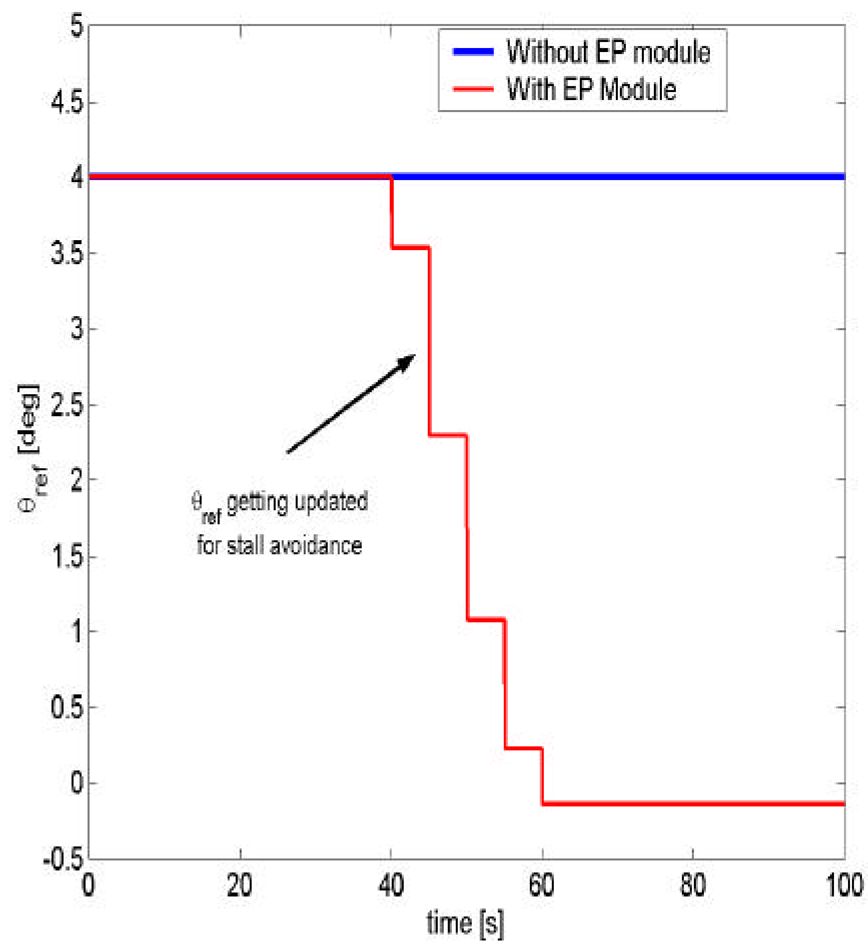


Without EP module
With EP Module
Pitching down to avoid
the angle of attack stall
limit violation
θ [deg]
time [s]
9
8
7
6
5
4
3
2
0
20
40
60
80
100



Without EP module
With EP Module
θref getting updated
for stall avoidance
θref [deg]
time [s]
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
20
40
60
80
100

Summary

- The pitch command inputs need to be reduced in case of icing to stay within the prescribed limit

- The EP module works fine with varying stall angle limits with icing

Work in Progress

- Test the steady state based envelope protection module in the simulator

- Complete study on alternative PAH A/P design